Scissors congruence K theory of manifolds

jt w/ Hoekzema, Murray, Rovi, Semikina

& Raptis, Semikina

<u>Classical question</u>: Given two polyhedra P, Q, when are they scissors cong. ?

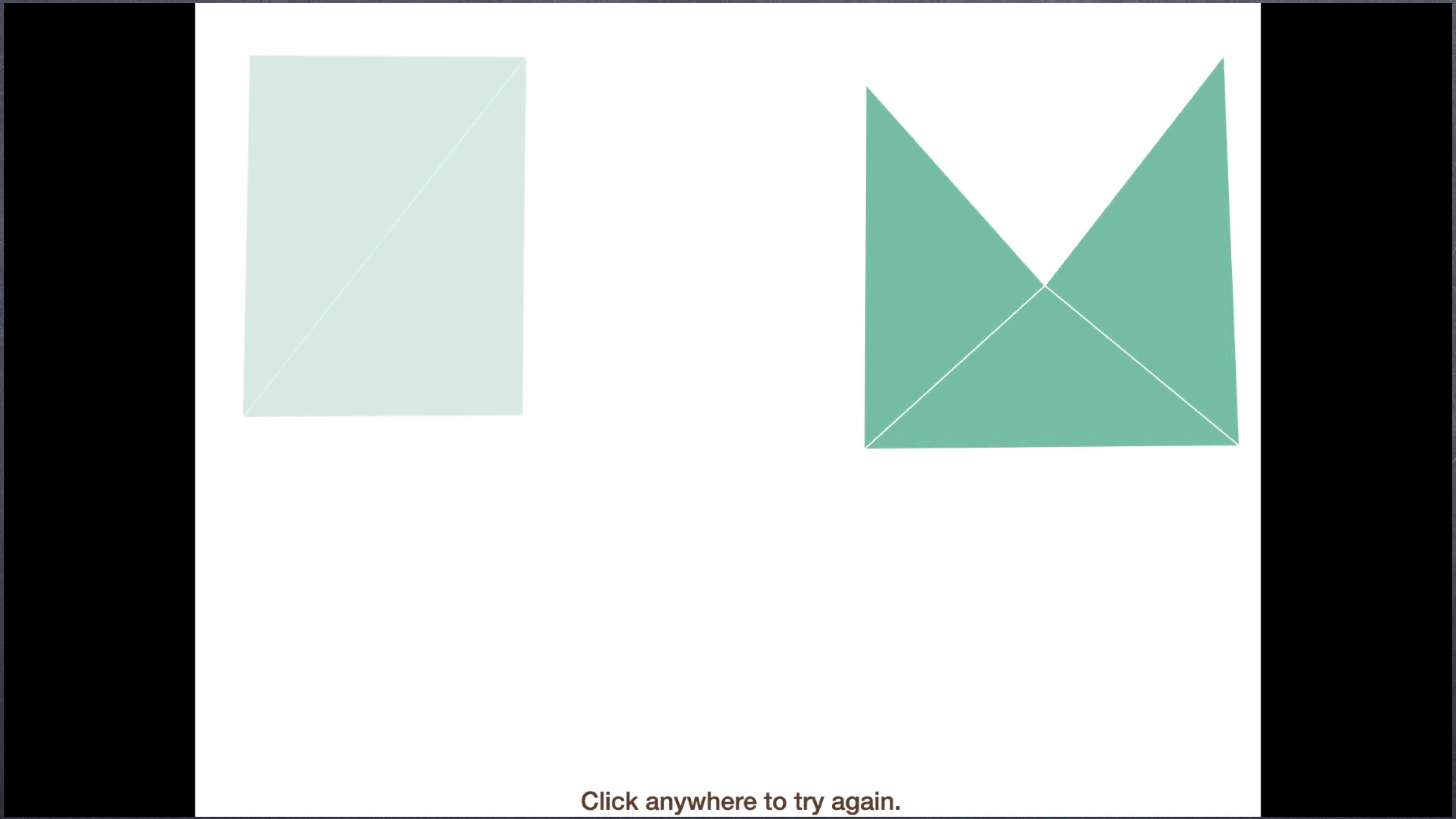
i.e. $P = \bigcup_{i=1}^{n} P_i$ $\quad Q = \bigcup_{j=1}^{n} Q_j$ $\quad P_i \cong Q_j$

P_i's Q_j's only intersect in edges & vertices

in 2D: need P, Q to have the same area.

<u>Q</u>: Is this the only SC invariant?

Example: (Wallace - Bolyai - Gerwien theorem)



Animation: Smirnov + Epstein on github

Q: What about dim 3?
(Hilbert's 3rd problem)

Thm: For 3D, the only SC invariants are
- volume
- Dehn invariant $\in \mathbb{R} \otimes \mathbb{R}/\pi\mathbb{Z}$

For arbitrary dim,
open question!

Q: What about dim 3? (Hilbert's 3rd problem)

Thm (Dehn, Sydler)

In 3D, only SC invariants are

- volume
- Dehn invariant $\in \mathbb{R} \otimes \mathbb{R}/\pi\mathbb{Z}$

For arbitrary dim n:

open question!

Scissors congruence for compact, oriented smooth manifolds

- start w/ a smooth manifold
- cut along a codim 1 submanifold w/ trivial normal bundle to obtain 2 disjoint pieces
- paste back along a diffeo of the boundaries

Def: $N \cup_\phi N' \underset{SK}{\simeq} N \cup_\psi N'$

"SK equivalent"

SK = "schneiden und kleben"
= "cut and paste"

(SC "scissors congruent")

Eg