

Reading: [Mal23, Section 3.2] and [Wei94, Section 10.2]

- (1) A spectrum X is *rational* if each of its homotopy groups is a $\mathbb{Q}$ -vector space. Let $\mathrm{ho}(\mathcal{S}p)_{\mathbb{Q}}$ be the full subcategory of $\mathrm{ho}(\mathcal{S}p)$ whose objects are rational spectra. Prove that $\mathrm{ho}(\mathcal{S}p)_{\mathbb{Q}}$ is a triangulated subcategory of $\mathrm{ho}(\mathcal{S}p)$.
- (2) Define a triangulated functor $H: \mathcal{D}(\mathbb{Q}) \rightarrow \mathrm{ho}(\mathcal{S}p)$ such that $\pi_n H(V_{\bullet}) = H_n(V_{\bullet})$ for all $n \in \mathbb{Z}$. *Hint: any chain complex of $\mathbb{Q}$ -vector spaces is quasi-isomorphic to its homology.*
- (3) Show that the class of stable equivalences is saturated in $\mathcal{S}p$.
- (4)
 - (a) Show that in a triangulated category $\mathcal{C}$, all monomorphisms are split monomorphisms.
 - (b) Show that the triangulated category $\mathcal{D}(\mathbb{Z})$ is not an abelian category.
 - (c) Show that $\mathrm{ho}(\mathcal{S}p)$ is not an abelian category.

REFERENCES

- [Mal23] Cary Malkiewich. Spectra and stable homotopy theory. http://people.math.binghamton.edu/malkiewich/spectra_book_draft.pdf, October 2023.
- [Wei94] Charles A. Weibel. *An introduction to homological algebra*, volume 38 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 1994.