

RING SPECTRA

NAME: _____

Reading: [Mal23, Sections 6.1 and 6.2].

- (1) Let $F \dashv G$ be an adjoint pair of functors between monoidal categories.
 - (a) Show that if F is strong symmetric monoidal, then G is lax symmetric monoidal.
 - (b) If G is lax symmetric monoidal, can you say anything about F ? Give a proof or counterexample.
- (2) Every spectrum X is an S -module. Describe the π_*S -action on π_*X .
- (3) Let R be a commutative ring spectrum.
 - (a) Show that the forgetful functor from the category of R -module spectra to symmetric (or orthogonal) spectra has a left adjoint.
 - (b) Let M be an R -module spectrum such that π_*M is free as a graded π_*R -module. Show that M is stably equivalent to a wedge sum of shifts of copies of R .

REFERENCES

[Mal23] Cary Malkiewich. Spectra and stable homotopy theory. http://people.math.binghamton.edu/malkiewich/spectra_book_draft.pdf, October 2023.