A Reading Course on Stable Homotopy Theory

Kiran Luecke

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This reading course is an introduction to stable homotopy theory. Stable homotopy theory is a diverse and rapidly growing field with connections to algebraic geometry, number theory, representation theory, and mathematical physics.

The following wonderful materials, consisting of problem sets, solutions, lecture outlines, and detailed references, are adapted from a course taught by David Mehrle through the electronic Computational Homotopy Theory online research community in Spring 2024 and Spring 2025. I am are extremely grateful to him.

Learning Outcomes

After taking this course, students will be able to engage in research projects in stable homotopy theory or related fields that use stable techniques. Students will also gain experience communicating mathematics by giving talks, writing, and discussing mathematics with peers.

Prerequisites

- Students should have taken the equivalent of two semesters of algebraic topology at the graduate level, including homology, cohomology, and homotopy groups π_n for $n > 1$. Exposure to topological K-theory, characteristic classes, cobordism, and other topics in algebraic topology is not required, although it may be helpful.
- Students should have a working knowledge of category theory, including but not limited to: natural transformations, limits, colimits, adjunctions, abelian categories, and symmetric monoidal categories.

Topics

The course covers the following topics in the following order:

(1) Homotopical Categories

(2) Homotopy Limits and Colimits

(3) Stable Phenomena in Spaces

(4) Generalized Cohomology Theories

(5) Sequential Spectra

(6) Stability Theorems

(7) Cellular and Finite Spectra

(8) The Stable Homotopy Category

(9) The Smash Product

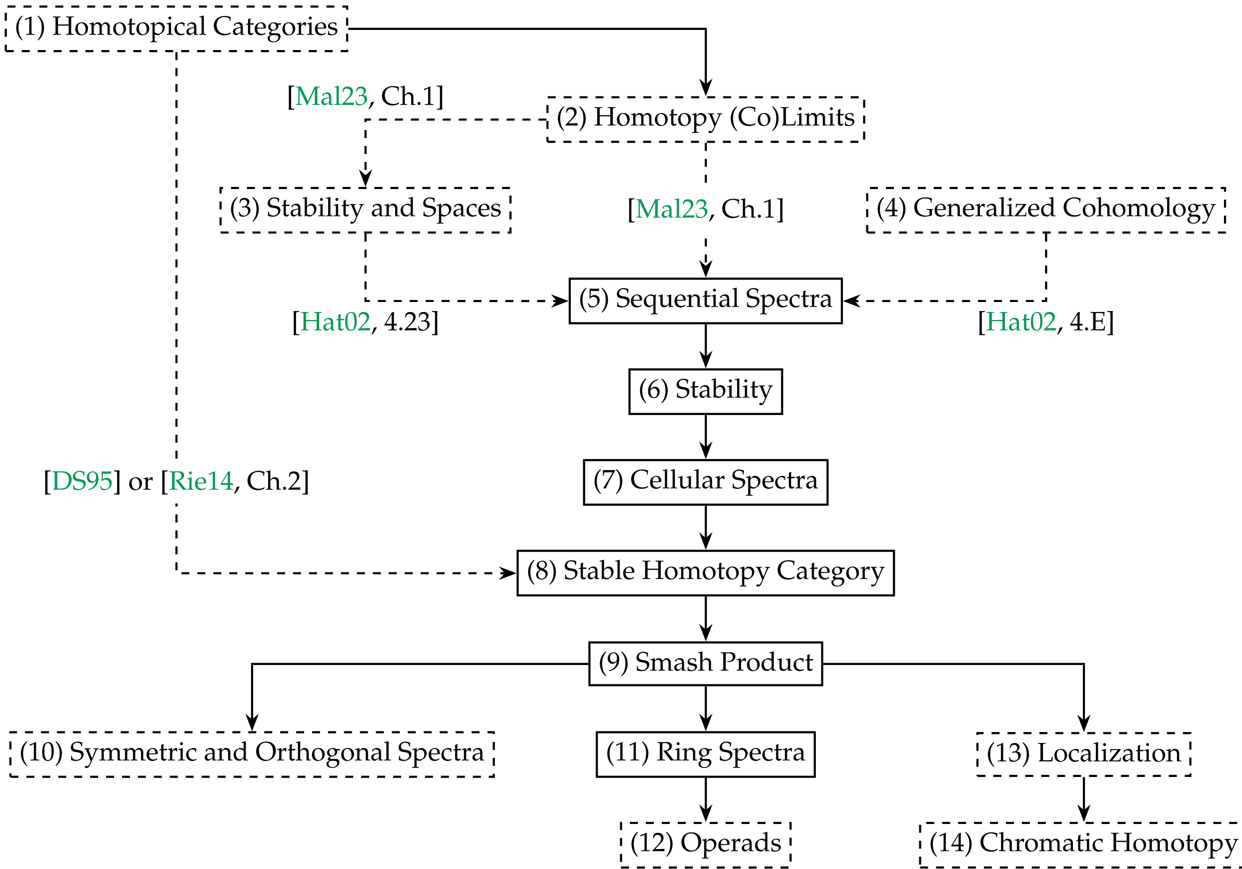
(10) Symmetric and Orthogonal Spectra

(11) Ring Spectra

(12) Operads

(13) Localization

(14) Chromatic Homotopy



A dependency tree for the topics in the course. Topics with solid borders form the core of the course; topics with dashed borders are optional. Solid arrows indicate mandatory prerequisites; dashed arrows indicate soft prerequisites that might be bypassed with other outside reading (suggested reading labels the arrow).

Logistics

The materials for this course consist of lecture outlines and problem sets (with solutions). The structure for this reading course is that we meet twice per week, tackling one topic per week. In the first meeting, one student uses the lecture outline to prepare and deliver a lecture. In the second meeting, all participants work on the problem sets together. A schematic schedule is displayed below.

Week 1: Topic A	Lecture Problem Session	(Meeting 1) (Meeting 2)
Week 2: Topic B	Lecture Problem Session	(Meeting 2) (Meeting 3)
Week 3: Topic C	Lecture Problem Session	(Meeting 4) (Meeting 5)
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Lecture Outlines

Each topic comes with a lecture outline detailing one possible path through the topic. The outlines suggest what to talk about, but they do not always contain proofs or detailed examples. Students will have to work to construct their own talk from the outlines provided.

These outlines are only suggestions. There are many different directions that one could go with each talk. Students should use the talks as an excuse to learn something they're curious about. The outlines might also be too much material for the time you have allocated. In that case, feel free to focus on the important things and cut other material. I am of course here to help with such decisions.

Problem Sets and Solutions

It is recommended that students not only solve the problems, but also write their solutions down, before reading the solutions. For a self-guided reading course, we recommend that students engage in peer grading of each other's written solutions to the problem sets. Writing clearly and communicating ideas to your peers is a core skill of mathematical research, and one which is eternally undervalued.

A final exam is also provided, should students want to challenge themselves at the end of the semester.

References

The references David consulted while constructing this course are listed below. Almost all of the material referenced is available for free online; students should not need to buy any resources to engage with this course.

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